Test II Solutions

MAT 203, Elementary Statistics, Term IV — Dowman P Varn, Instructor
Coker College — Test Date: 09 April 2009

Please complete the following problems. You may use a $4'' \times 6''$ formula sheet, a calculator and paper and pencil, but you may not use your text or any other resources. You must work alone. Please write legibly. **Please clearly indicate how you do each problem.** This test counts 20% toward your final grade. Good Luck!

1. Tonya shuffles a standard deck of 52 cards. She turns over the the top two cards and finds that they are the K♣ and the 8♠. (a) What is the likelihood that the third card in the deck is the 5♥? (b) What is the likelihood instead that the third card is either an Ace or a ♠?

(a) There are fifty cards remaining in the deck, and each card is equally likely to be next. Thus $P(5\heartsuit) = \frac{1}{50}$.
(b)

$$\begin{aligned} P(\text{A } or \spadesuit) &= P(\text{A}) + P(\spadesuit) - P(\text{A } and \spadesuit) \\ &= \frac{4}{50} + \frac{12}{50} - \frac{1}{50} \\ &= \frac{15}{50} \\ &= 0.30 \end{aligned}$$

2. Calculate ${}_8C_3$.

$${}_8C_3 = \frac{8!}{(8-3)! \times 3!} = \frac{8!}{5! \times 3!} = 56$$

3. A car towing service company averages two calls per hour. Use the Poisson distribution to find the probability that in a randomly selected hour the number of calls is three.

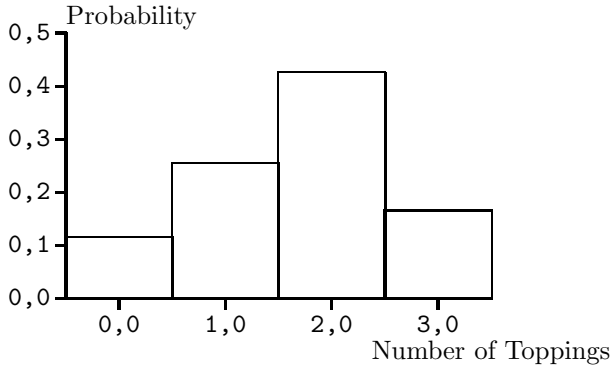
We have $\mu = 2$ and $x = 3$. Using the statistical function `poissonpdf(2,3)` on the TI-83 we find $P(3) = 0.180$.

4. Paula's Pizza Parlour makes pizzas with none, one, two or at most three toppings. Table 1 gives the average number of pizzas made in each month with each number of toppings. (a) Draw the probability distribution, and (b) find the probability that a randomly chosen pizza has two or more toppings.

Number of Toppings	Number of Pizzas	Probability
0	121	0.151
1	204	0.255
2	340	0.425
3	135	0.169

Table 1: Number of toppings and the number of pizzas made with that number of toppings.

(a) The probabilities are calculated in the third column of Table 1 and a histogram the probability distribution is shown below.



(b)

$$\begin{aligned} P(2\,or\,3) &= P(2) + P(3) \\ &= 0.425 + 0.169 \\ &= 0.594 \end{aligned}$$

5. What is the area under the standard normal curve to the left of $z = -0.4$?

The area to the left is just that area over the interval $(-\infty, -0.4)$. Using `normalcdf(-100, -0.4)` on the TI-83, we find the area is 0.345.

6. Assume that the salaries of elementary school teachers are normally distributed with a mean of $36,000 and a standard deviation of $4000. What salary separates the top 10% of teachers from the rest? (ie, What salary represents the 90th percentile?)

This is most easily found using the `invNorm` function on the TI-83. Identifying $\mu = \$36{,}000$ and $\sigma = \$4000$, we find, `invNorm(0.90, 36000, 4000)` = $41,100.

7. The mean age of employees at a large corporation is 51.5 years, with a standard deviation of 4.0 years. Random samples of size 64 are drawn from this population and the mean of each sample is determined. Use the Central Limit Theorem to find: (a) the mean of the means of the sample distributions, and (b) the standard error of the mean.

(a) The mean of the means of the sampling distributions is the same as that of the population, so $\mu_{x_{\text{mean}}} = \mu = 51.5$ years.

(b) The standard error of the mean is just $\sigma_{x_{\text{mean}}} = \frac{\sigma}{\sqrt{n}} = \frac{4.0}{\sqrt{64}} = 0.5$ years.

8. For the distribution in Table 2, find (a) the mean and (b) the standard deviation.

(a) Using the statistical functions on the TI-83, we find $x_{\text{mean}} = 23.4$.

(b) Similarly, we find $s = 7.92$.

9. For the distribution in Table 2, find the 95% confidence interval about the mean.

Using the `ZInterval` function on the TI-83, we find (20.566, 26.234).

20	12	30	29	42	28	22	32	15	17
33	22	20	19	22	18	32	33	13	19
11	15	35	21	22	11	25	31	27	26

Table 2: A sample of numbers drawn randomly from some population. You do not know the nature of the distribution of the population.

10. Suppose instead that there were only fifteen data points in Table 2 and the population from which they were drawn was normally distributed. What distribution would you use now to estimate the 95% confidence interval?

You should use the t-distribution.