Test I Solutions

MAT 203, Elementary Statistics, Term IV
Coker College

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Please complete the following problems. You may use a $4'' \times 6''$ formula sheet, a calculator and paper and pencil, but you may not use your text or any other resources. You must work alone. Please write legibly. **You must show your work in order to receive full credit.** Please write out any formulae that you use so that I can understand how you did the problem. You may, of course, check your answers using the statistical functions on your calculator. This test counts 15% toward your final grade. Good Luck!

1. Two hundred students take a test. The mean score on the test is 70 out of 100 with a standard deviation of 10. You do not know the distribution of the scores. Nevertheless, calculate the minimum number of students that have scores that are between 50 and 90.

Notice that if the standard deviation is 10, and the scores range from 50 to 90, centered about the mean of 70, this covers a range of two standard deviations about the mean. We don't know the distribution, but *Chebychev's Theorem is valid for any distribution.* Let F be the minimum fraction of data points within two standard deviations of the mean.

$$F = \left(1 - \frac{1}{k^2}\right) = \left(1 - \frac{1}{2^2}\right) = \left(1 - \frac{1}{4}\right) = \frac{3}{4}$$

So the minimum number of scores or data points within this range is just $200 \times \frac{3}{4} = 150$.

2. Now suppose that the distribution of scores for the above problem is a normal or bell-shaped distribution. About how many students scored between 50 and 90?

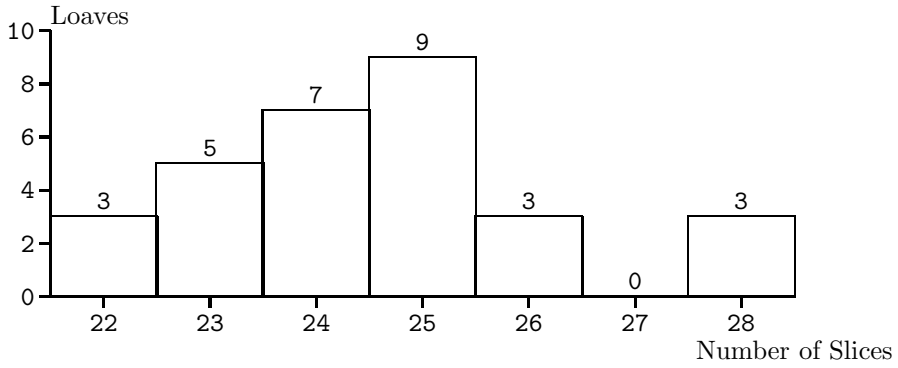
If the distribution is bell-shaped, we can just use the Empirical Rule: About 95% of the data points are within two standard deviations of the mean. So, if there are 200 students, then $200 \times \frac{95}{100} = 190$ students are within two standard deviations of the mean.

(For problems 3 - 10) Bob the Baker is notoriously picky about his bread. He is concerned that when his employees slice his fresh baked loaves, they don't cut each loaf into the same number of slices each time. He wants measure the severity of the problem. He chooses at random 30 loaves off his shelf. He then counts the number of slices in each loaf. His results are shown in Table 1.

3. Calculate the relative frequency of finding 26 slices in a loaf.

Three loaves out of thirty have 26 slices, so the relative frequency f_{relative} of 26 slices is just $f_{\text{relative}} = \frac{3}{30} = \frac{1}{10}$ or 10%.

4. Draw a histogram of this distribution.



5. What is the mode of this distribution?

   The mode of a distribution is just that value of the distribution that appears with the greatest frequency. From Table 1, loaves with 25 slices appear more often than any of the others, so $x_{\text{mode}} = 25$ slices.

6. What is the mean of this distribution?

$$\begin{aligned} x_{\text{mean}} &= \frac{1}{n}\sum_i f_i^{\text{class}} x_i^{\text{class}} \\ &= \frac{1}{30}\left[(3\times 22)+(5\times 23)+(7\times 24)+(9\times 25)+(3\times 26)+(0\times 27)+(3\times 28)\right] \\ &= \frac{1}{30}\left[66+115+168+225+78+0+84\right] \\ &= \frac{736}{30} \\ &= 24.533 \text{ slices} \end{aligned}$$

7. What is the median of this distribution?

   Since there are an even number of data points in the distribution, we need to average the center-most two.

$$x_{\text{median}} = \frac{x_{15}+x_{16}}{2} = \frac{24+25}{2} = 24.5 \text{ slices}$$

8. What is the standard deviation of this distribution?

$$\begin{aligned} s &= \sqrt{\frac{\sum_i f_i^{\text{class}}(x_i^{\text{class}} - x_{\text{mean}})^2}{n-1}} \\ &= \sqrt{\frac{1}{30-1}} \times [3\times(22-24.533)^2 + 5\times(23-24.533)^2 + 7\times(24-24.533)^2 \\ &\quad +9\times(25-24.533)^2 + 3\times(26-24.533)^2 + 0\times(27-24.533)^2 + 3\times(28-24.533)^2]^{\frac{1}{2}} \\ &= \sqrt{\frac{1}{29}} \times [19.25+11.75+1.99+1.96+6.46+0+36.06]^{\frac{1}{2}} \\ &= 1.63 \text{ slices} \end{aligned}$$

9. Calculate the range of this distribution.

   The range R is just given by:

$$R = x_{\max} - x_{\min} = 28 - 22 = 6 \text{ slices}$$

10. Calculate the interquartile range for this distribution.

    The interquartile range IQR is given by $\text{IQR} = Q_3 - Q_1$. So one must first find Q_3 and Q_1. Divide the data set into two halves and find the median of each half. Q_1 is the median of the first half and Q_3 is the median of the second half. The median of the first half is 23 slices and the median of the second half is 25 slices. So the interquartile range IQR is given by $\text{IQR} = Q_3 - Q_1 = 25 - 23 = 2$ slices.

Number of Slices per Loaf	Number of Loaves of Bread
22	3
23	5
24	7
25	9
26	3
27	0
28	3

Table 1: The first column shows the number of slices found in a particular loaf of bread, and the second column gives the number of loaves with that many slices.