Homework Set 2 Solutions

MAT 203, Elementary Statistics, Term IV
Coker College
Dowman P Varn, Instructor

Homework Policy: The point of homework is to learn by doing. I have no problem if you work in groups, use the internet or use human resources to help you complete the assignment. I do ask that you not copy someone else's homework. Please do all the mathematics yourself. By submitting your homework, you implicitly signify that it is your own. This assignment is due at the beginning of class on 24 March 2009.

1. Consider two six-sided dice. Each face on each dice has a 1-in-6 chance of coming up when the die is rolled, and the faces are numbered one through six. Susan rolls a single die and it comes up five. What is the outcome of this trial? What is the sample space?

> The outcome of a trial is just the result, in this case 5. The sample space is the set of all possible outcomes $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$.

2. Most credit card numbers are now 16 digits. The first four are used to identify the credit card issuer. Assume that the other 12 are unique to each card. How many possible distinct cards can be issued by a company? Compare this to the number of people on the Earth.

> Each digit can range from zero to nine, and each digit is independent of the others. So for twelve digits, we have 10^{12} possible unique combinations. There are about 7×10^9 people on the Earth, so each issuer can give each man, woman and child on the planet 142 unique credit card numbers.

3. Susan now rolls a pair of dice. She would like to roll a total of four or less. What is the probability of doing this?

> The probability of rolling a four or less is just $P(4\,\text{or less}) = P(2) + P(3) + P(4)$. There is one way to roll a 2, and that is $(1, 1)$. To get a 3, one can roll $(1, 2)$ or $(2, 1)$, for a total of two ways. To get a four, one can roll $(1, 3)$, $(2, 2)$ or $(3, 1)$, for a total of three ways. There are $6 \times 6 = 36$ possible outcomes, each equally likely. So $P(4\,\text{or less}) = P(2) + P(3) + P(4) = \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{1}{6}$.

4. Suppose that the Susan rolls the dice one at a time. The first die comes up one. She then rolls the second die. What is the probability that it is a three?

> Trials are independent, so the result of the first trial has no bearing on the result of the second. There are six possible outcomes and each is equally likely, so $P(3) = \frac{1}{6}$.

5. Problem 13, p. 142 of Larson and Farber. An insurance company is hiring for two positions: an actuary and a claims adjuster. How many ways can these positions be filled if there are 9 people applying for the actuary position and 15 people applying for the claims adjuster position?

> Let us assume that a person can apply for only one job. Since the choice of a person for each job is independent of who is chosen for the other, there are $9 \times 15 = 135$ possible ways for these positions to be filled.

6. I have a standard deck of 52 cards. (See page 136 of Larson and Farber.) What is the probability that a randomly selected card is a spade? So suppose I remove two of the spades from the deck. What is the probability now that a randomly selected card is heart?

> There are 13 spades in a deck of 52 cards. Each card is equally likely to be selected. So $P(\spadesuit) = \frac{13}{52} = \frac{1}{4} = 0.25$. If I remove two of the spades from the deck, the deck now has 50 cards, and 13 of them are hearts. Again, if each card is equally likely to be chosen, $P(\heartsuit) = \frac{13}{50} = 0.26$.

7. Problem 28, p. 158 of Larson and Farber. A printing company's bookbinding machine has a probability of 0.005 of producing a defective book. The machine is used to bind three books. (a) Find the probability that none of the books is defective. (b) Find the probability that at least one of the books is defective. (c) Find the probability that all of the books are defective.

(a) Each binding is independent, so the probability of three consecutive correctly bound books, $P(3\text{s})$ is just $P(3\text{s}) = P(1\text{s}) \times P(1\text{s}) \times P(1\text{s}) = (0.995)(0.995)(0.995) = 0.985$.

(b) If at least one is defective, this is the complement of (a), so we have $P(\text{at least one defective}) = 1 - P(3\text{s}) = 1 - 0.985 = 0.015$.

(c) Again, each bind is independent, and we want to find the probability of three consecutive defective bindings, $P(3\text{u})$. $P(3\text{u}) = P(1\text{u}) \times P(1\text{u}) \times P(1\text{u}) = (0.005)(0.005)(0.005) = 1.25 \times 10^{-7}$.

8. What is the probability of selecting either a face card (K, Q, J) or a club from a deck of cards?

Choosing a face card or a club from a deck are not independent events. Let $P(\clubsuit)$ be the probability of choosing a club and $P(\text{FC})$ be the probability of choosing a face card. Now, from problem 6, $P(\clubsuit) = 0.25$. Since there are are twelve face cards, $P(\text{FC}) = \frac{12}{52} = \frac{3}{13} = 0.231$. Now the probability of finding a card that is both a club *and* a face card is just given by the number of face cards that are clubs (which would be three, K$\clubsuit$, Q$\clubsuit$ and J$\clubsuit$) divided by the number of cards, so $P(\clubsuit\,\text{and}\,\text{FC}) = \frac{3}{52} = 0.058$. Thus,

$$\begin{aligned} P(\clubsuit\,\text{or}\,\text{FC}) &= P(\clubsuit) + P(\text{FC}) - P(\clubsuit\,\text{and}\,\text{FC}) \\ &= 0.25 + 0.231 - 0.058 \\ &= 0.423 \end{aligned}$$

9. Problem 15, p. 166 of Larson and Farber. A company that makes cartons finds the probability of producing a carton with a puncture is 0.05 ($P(\text{p}) = 0.05$), the probability that a carton has a smashed corner is 0.08 ($P(\text{c}) = 0.08$) and the probability that a carton has a puncture and has a smashed corner is 0.004 ($P(\text{p and c}) = 0.004$). (a) Are the events "selecting a carton with a puncture" and "selecting a carton with a smashed corner" mutually exclusive? Explain. (b) If a quality inspector randomly selects a carton, find the probability that the carton has a puncture or has a smashed corner.

(a) Since a carton can have both a puncture and a smashed corner, as demonstrated by the fact that $P(\text{p and c}) \neq 0$, the events are not mutually exclusive.

(b)

$$\begin{aligned} P(\text{p or c}) &= P(\text{p}) + P(\text{c}) - P(\text{p and c}) \\ &= 0.05 + 0.08 - 0.004 \\ &= 0.126 \end{aligned}$$

10. Calculate ${}_6P_2$.

$${}_6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = 30$$

11. How many possible orderings are there of a deck of 52 cards?

There are $52! \approx 8 \times 10^{67}$ possible orderings.

12. How many possible hands of five cards can one draw from a standard deck of 52 cards?

This is just a combinatorics problem. We need ${}_{52}C_5 = \frac{52!}{(52-5)! \times 5!} = \frac{52!}{47! \times 5!} = 2,598,960$.

13. Problem 20, p. 178 of Larson and Farber. Eight people compete in a downhill ski race. Assuming there are no ties, in how many different orders can the skiers finish.

This is a permutations problem. The total number of ways is $8! = 40,320$.

14. Problem 25, p. 179 of Larson and Farber. A landscaper wants to plant four oak trees, eight maple trees, and six poplar trees along the border of a lawn. The trees are to be spaced evenly apart. In how many distinguishable ways can they be planted?

So this a distinguishable permutations problem. The total number of distinguishable ways is $\frac{18!}{4!\times 8!\times 6!} = 9,189,180$.

15. Problem 40, p. 180 of Larson and Farber. In a certain state, each automobile license plate number consists of two letters followed by a four-digit number. How many distinct license plate numbers can be formed if (a) there are no restrictions and (b) the letters O and I are not used. What is the probability of selecting at random a license plate that ends in an even number?

(a) For no restrictions, we have $N = 26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6,760,000$.

(b) If O and I are not uses, $N = 24 \times 24 \times 10 \times 10 \times 10 \times 10 = 5,760,000$.

(c) Since half the numbers are even, $P(\text{even}) = \frac{1}{2}$.

16. Calculate ${}_7C_3$.

$$
{}_7C_3 = \frac{7!}{(7-3)! \times 3!} = \frac{7!}{4! \times 3!} = 35
$$

Extra Credit. A woman has two children. (Consider the probability of giving birth to either a boy or a girl to be 50%). She tells you that one of them is a girl. What is the probability that the other child is also a girl?

This one is a little difficult. So, there are four possible combinations of two children with two genders. Let b stand for boy and g stand for girl. The possible combinations are (b,b), (b,g), (g,b) and (g,g). Generally, each is equally likely. But if one of the children must be a girl, then the probability of two boys must be zero. The remaining three are equally likely, namely each has a probability of $\frac{1}{3}$. There is only one combination that has two girls, and it has a probability of $\frac{1}{3}$.